theory should also be possible for concave surface flows with leading-edge bluntness.

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Solution of Lateral Vibration of a Cantilever by Parameter Differentiation

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A NONITERATIVE method for the solution of nonlinear algebraic equations has recently been developed by Kane[1] and Yakolev.[2] Briefly, the solution of a nonlinear algebraic equation

$$F(x) = 0 \tag{1}$$

can be obtained as follows: First, an arbitrary first approximation of the solution of Eq. (1), say x_I, is chosen. A new equation is then defined as

$$F(x) = F(x_I)\ (1 - \tau) \tag{2}$$

where τ is a parameter which changes from 0 to 1. When τ equals to zero, the solution of Eq. (2) becomes x_I, the first approximation of the solution of Eq. (1). When τ equals 1, Eq. (2) is reduced to Eq. (1), which means the solution of Eq. (2) becomes the solution of Eq. (1). For this reason, the solution of Eq. (2), as τ changes from 0 to 1, is expected to change from x_I to the solution of Eq. (1). This points out the need of a differential equation of $dx/d\tau$, originating from Eq. (2). Such an equation can be obtained by differentiating Eq. (2) with respect to τ. We therefore get

$$F'(x)\,dx/d\tau = -F(x_I) \tag{3}$$

subject to the initial condition

$$\tau = 0:\ x = x_I$$

By integrating Eq. (3) from 0 to 1, the solution of Eq. (3) will be changed from x_I (at $\tau = 0$) to the solution of Eq. (1) (at $\tau = 1$). The merit of the method is that it eliminates the iteration process. It should be noted that Eq. (3) is a variation of the interative Newton-Raphson method,

$$x_{k+1} = x_k - F(x_k)/F'(x_k)$$

As a result of multiplicity of solutions for nonlinear algebraic equations, the specific solution approached will depend on the value of x_I selected. While this property might pose difficulties for certain problems in the selection of x_I where only *one* of the solutions is physically meaningful, it can be used to the advantage of the engineer for other problems where this method can be used to systematically search for all possible solutions. An example will be given in this Note to illustrate this particular feature of the method.

Consider a uniform Bernoulli-Euler beam of length L clamped at one end and subjected at the other end to a constant tensile follower force P, as shown in Fig. 1. The amplitude of s small transverse vibration about the x-axis is governed by the differential equation[3]

$$EI\frac{\partial^4 y}{\partial x^4} - P\frac{\partial^2 y}{\partial x^2} + \rho\frac{\partial^2 y}{\partial t^2} = 0 \tag{4}$$

where E, I, and ρ are, respectively, the Young's modulus, the moment of inertia, and the density. By introducing the dimensionless variables

$$\xi = \frac{x}{\ell}\ .\quad \tau = t\left(\frac{EI}{\rho\ell^4}\right)^{1/2},\quad k^2 = \frac{P\ell^2}{EI} \tag{5}$$

Eq. (4) becomes

$$(\partial^4 y/\partial\xi^4) - k^2(\partial^2 y/\partial\xi^2) + \partial^2 y/\partial\tau^2 = 0 \tag{6}$$

Following Anderson and King,[3] the solution is written in the form:

$$y(\xi,\tau) = Y(\xi)\sin\omega\tau \tag{7}$$

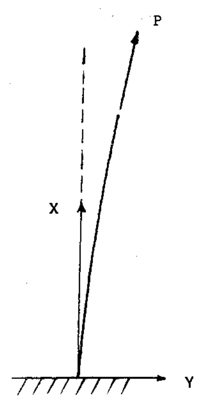

Fig. 1 The cantilever.

Received December 5, 1974; revision received March 21, 1975.
Index categories: Aircraft Structural Design (including Loads).

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with the boundary conditions for $Y(\xi)$ given by

$$Y(0)=Y'(0)=Y''(1)=Y'''(1)=0$$

Substituting the solution (7) into Eq. (6), the following nonlinear algebraic equation is obtained:

$$k^4+2\omega^2(1+\cosh\lambda_2\cos\lambda_1)-2\omega k^2\sinh\lambda_2\sin\lambda_1=0 \quad (8)$$

where

$$\lambda_1=[(\omega^2+k^4/4)^{1/2}-k^2/2]^{1/2}$$

and

$$\lambda_2=[(\omega^2+k^4/4)^{1/2}+k^2/2]^{1/2}$$

Equation (8) is a nonlinear algebraic equation for the solution of the natural frequency ω. For a given value of k^2, Eq. (8) will yield a family of values of ω's corresponding to the various natural frequencies. This is seen to be an equation with multiple solutions where all the solutions are physically meaningful. Anderson and King[3] gave the first three natural frequencies in their work. We will now apply the method of parameter differentiation to systematically search for the multiple solutions.

The differential equation corresponding to Eq. (3) for the solution of Eq. (8) is

$$\Big\{4\,\omega(1+\cosh\lambda_2\cos\lambda_1)-k^2\sinh\lambda_2\sin\lambda_1 + \frac{\omega^2}{(\omega^2+k^4/4)^{1/2}}\left[\frac{\sinh\lambda_2\cos\lambda_1}{\lambda_2}-\frac{\cosh\lambda_2\sin\lambda_1}{\lambda_1}\right] - \frac{\omega^2k^2}{2(\omega^2+k^4/4)^{1/2}}\left[\frac{\cosh\lambda_2\sin\lambda_1}{\lambda_2}+\frac{\sinh\lambda_2\cos\lambda_1}{\lambda_1}\right]\Big\}\frac{d\omega}{d\tau}$$
$$=-[k^4+2\omega_0^2(1+\cosh\lambda_{20}\cos\lambda_{10}) - \omega_0k^2\sinh\lambda_{20}\sin\lambda_{10}] \quad (9)$$

Table 1 Solutions of $\omega(\tau)$ for various values of ω_0 ($k^2=20$)

	$\omega=1$		$\omega=5$		$\omega=10$
τ	$\omega(\tau)$	τ	$\omega(\tau)$	τ	$\omega(\tau)$
0.0	1.0000	0.0	5.0000	0.0	10.0000
0.2	1.2409	0.2	4.5996	0.2	9.2058
0.4	1.4405	0.4	4.1398	0.4	8.2707
0.6	1.6141	0.6	3.5934	0.6	7.1022
0.8	1.7695	0.8	2.9047	0.8	5.4592
1.0	1.9112	1.0	1.9112	1.0	1.9112
	$\omega=25$		$\omega=40$		$\omega=50$
τ	$\omega(\tau)$	τ	$\omega(\tau)$	τ	$\omega(\tau)$
0.0	25.000	0.0	40.000	0.0	50.000
0.2	25.583	0.2	38.626	0.2	47.432
0.4	26.066	0.4	37.030	0.4	44.697
0.6	26.483	0.6	35.070	0.6	41.516
0.8	26.853	0.8	32.392	0.8	37.250
1.0	27.187	1.0	27.184	1.0	27.182
	$\omega=67$		$\omega=80$		$\omega=100$
τ	$\omega(\tau)$	τ	$\omega(\tau)$	τ	$\omega(\tau)$
0.0	67.000	0.0	80.000	0.0	100.000
0.2	67.387	0.2	78.572	0.2	96.455
0.4	67.749	0.4	76.929	0.4	92.605
0.6	68.089	0.6	74.964	0.6	88.079
0.8	68.410	0.8	72.451	0.8	81.996
1.0	68.716	1.0	68.714	1.0	68.712

where

$$\lambda_{10}=[(\omega_0^2+k^4/4)^{1/2}-k^2/2]^{1/2}$$

$$\lambda_{20}=[(\omega_0^2+k^4/4)^{1/2}+k^2/2]^{1/2}$$

and the initial condition is

$$\tau=0: \quad \omega(0)=\omega_0$$

Equation (9) is integrated by using Runge-Kutta method with $\Delta\tau=0.005$. To illustrate the procedure, let us consider the case in which $k^2=20$. By assigning ω_0 equal to $n\Delta\omega_0$, where n is taken to be 1,2, 3,..., successively, the solutions will naturally approach the individual natural frequencies, as demonstrated in Table 1. The first three cases in Table 1 (ω_0 =1,5,10) lead to the same natural frequency, $\omega=1.911$, even though their initial assumptions are different. Similar comments can be made relative to the other six cases. Namely, case 4-6 all approach to $\omega=27.18$ and cases 7-9 all approach to $\omega=68.71$, respectively. If higher natural frequencies are required, we simply continue to process by systemmatically using the values of ω_0 and the solutions of Eq. (9) at $\tau=1$ will naturally approach to the various natural frequencies.

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Experiment for Evaluation of Acceleration Measurement Capability

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DEVELOPMENT and qualification tests subjecting re-entry vehicles to impulse loads usually include measurements of acceleration response of internal components. Prior to testing, analytical predictions are made of the component responses to be measured. In general, complexity of structure and difficulties in representing load paths to components lead to analytical results of low credibility. In typical full-scale tests, records show negligible resemblance to the predictions, and frequently appear hopelessly contaminated with mechanical or electrical noise. The analyst and the component designer find themselves in the predicament of trying to establish component design or test criteria by reconciling predictions and test data in which they possess little faith.

To estimate a confidence level for certain component acceleration measurements, some experiments were performed using a simple structure having predictable response. The simple structure chosen was an aluminum ring comparable in structural response behavior to a vehicle cross section. A

Received January 20, 1975. This work was supported by the U. S. Atomic Energy Commission. The authors thank H. S. Tessler for performing the experiments.

Index category: Structural Dynamic Analysis.

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